B.Math.(Hons.) IInd year
Backpaper examination
Second semester 2012
Algebra IV
Instructor — Bharath A.Sethuraman
Answer any FIVE questions.

1. Let K be a field and let L be a normal extension of K. Let p be an irreducible polynomial in $K[X]$. Prove that all irreducible factors of p in $L[X]$ have the same degree.

2. Let K, L be (finite) Galois extensions of a field F. Prove that the composite KL is Galois over F.

3. Let L/K be a Galois extension of degree 100. Show that there is an intermediate field $K \subset E \subset L$ such that $[E : K] = 20$.

4. Prove that two finite fields of the same cardinality must be isomorphic.

5. Let p be an odd prime and let ω be a primitive p-th root of unity. Show that the norm $N_{\mathbf{Q}(\omega)/\mathbf{Q}}$ of $1 - \omega$ is p.

6. Prove that the angle of 40 degrees is not a constructible angle.

7. Let L, K fields be a finite extension. If A is an intermediate ring $K \subset A \subset L$, show that A is a field.

8. Let K be a field extension of F, let $\alpha \in K$ be algebraic over F, and let $t \in K$ be transcendental over F. Show that min $(F, \alpha) =$ min $(F(t), \alpha)$.